Maths for CS – Assignment 2b

Module Code 55-402612

The function f: N → N is defined recursively by the formula

1. Use the above definition to calculate f(1), f(2), f(3), f(4), and f(5).
2. f(1) = 1 = 1
3. f(2) = 3f(2-1)-1 = 3f(1)-1 = 3(1)-1 = 3-1 = 2
4. f(3) = 3f(3-1)-1 = 3f(2)-1 = 3(2)-1 = 6-1 = 5
5. f(4) = 3f(4-1)-1 = 3f(3)-1 = 3(5)-1 = 15-1 = 14
6. f(5) = 3f(5-1)-1 = 3f(4)-1 = 3(14)-1 = 42-1 = 41

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| n | 1 | 2 | 3 | 4 | 5 |
| f(n) | 1 | 2 | 5 | 14 | 41 |

1. Prove by induction that

for all n ∈ N.

1. Check that it works for the base case:
2. Assume that

for some

1. We want
2. Another function g: N → Z is defined by g(n) = 2n − 1. State (g ◦ f)(n) for n ∈ N.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| n | 1 | 2 | 3 | 4 | 5 |
| f(n) | 1 | 2 | 5 | 14 | 41 |
| (g ◦ f)(n) | 1 | 3 | 9 | 27 | 81 |

1. A further function h: Z → R is defined by h(n) = n2. State (h ◦ g)(n) for n ∈ N.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| n | 1 | 2 | 3 | 4 | 5 |
| f(n) | 1 | 2 | 5 | 14 | 41 |
| (g ◦ f)(n) | 1 | 3 | 9 | 27 | 81 |
| (h ◦ g)(n) | 1 | 9 | 81 | 729 | 6561 |

1. Hence, show that ((h ◦ g) ◦ f) = (h ◦ (g ◦ f)).

Let n = 3

f(n) = 5

h o g(f(n)) = 95-1 = 6561

g o f(n) = 81

h(g o f(n)) = 812 = 6561